

Advanced Algorithms

March 10, 2026

Logistics

- Exercise set due Thursday on gradescope
- Will put out Assignment 2 on Thursday as well
- Office hours this week:
 - Thursday before class instead
 - 10:30 – 11:30am

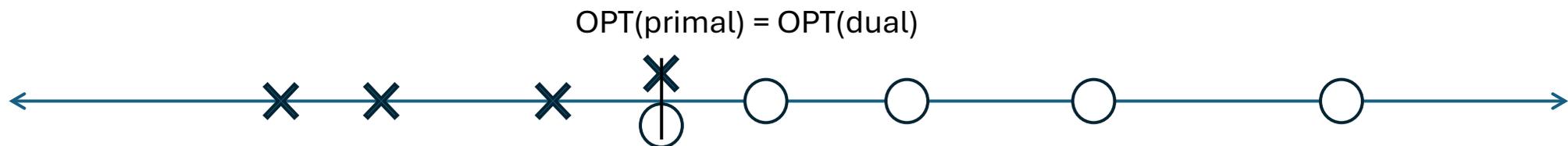
Last time: Duality

All Linear Programs come in dual pairs. One maximizes one minimizes.

Weak Duality: given a primal LP and its dual LP:

objective value of any
feasible solution to the
maximization problem \leq objective value of any
feasible solution to the
minimization problem

Strong Duality: if primal has an optimal solution, then so does its dual, and their optimal values are **equal**.



Consider the following

Given a **bipartite graph** $G = (L \cup R, E)$, define the following Linear Program:

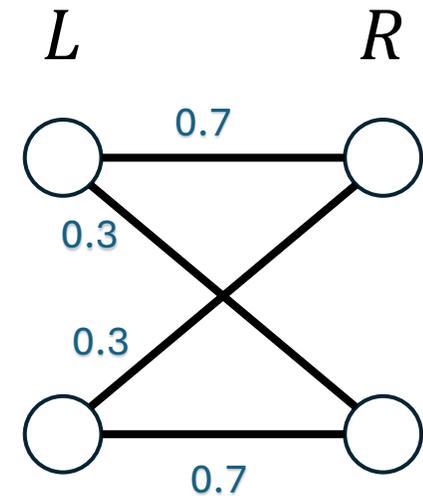
$$\min \sum_{e \in E} c_e x_e$$

such that:

$$\sum_{e \in \delta(u)} x_e = 1 \quad \text{for all } u \in L$$

$$\sum_{e \in \delta(v)} x_e = 1 \quad \text{for all } v \in R$$

$$x_e \geq 0 \quad \text{for all } e \in E$$



What are **integer solutions** to this LP? Think about what its **dual** would be.

Where we are

1. Linear Prog. can model many interesting problems, and is in P
2. Integer Prog. can model MANY problems but is NP-hard
3. We can use Linear Prog. to understand certain Integer Programs

*Also, a *language* of optimization

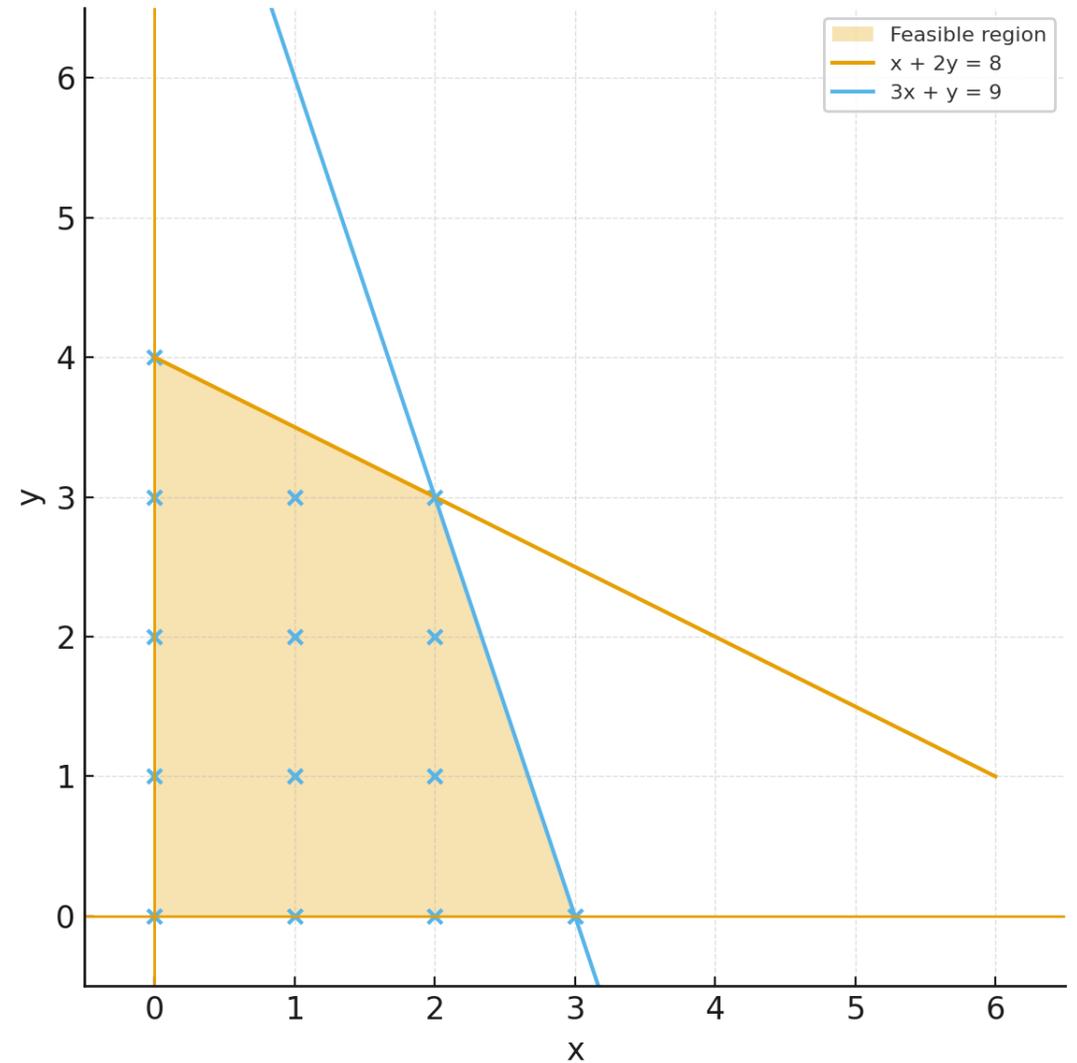
Where we are

1. Linear Prog. can model many interesting problems, and is in P
2. Integer Prog. can model MANY problems but is NP-hard
3. We can use Linear Prog. to understand certain Integer Programs

*Also, a *language* of optimization

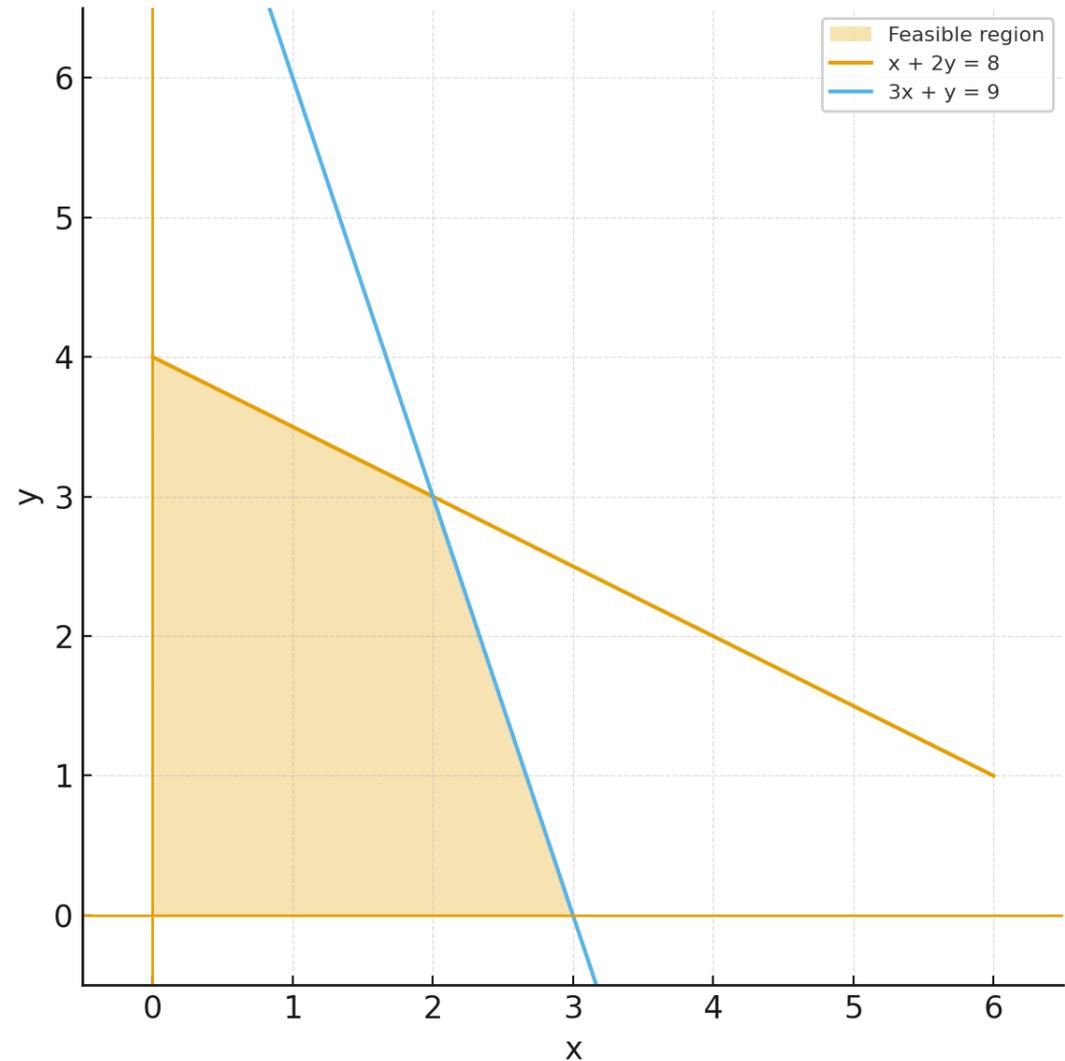
Today

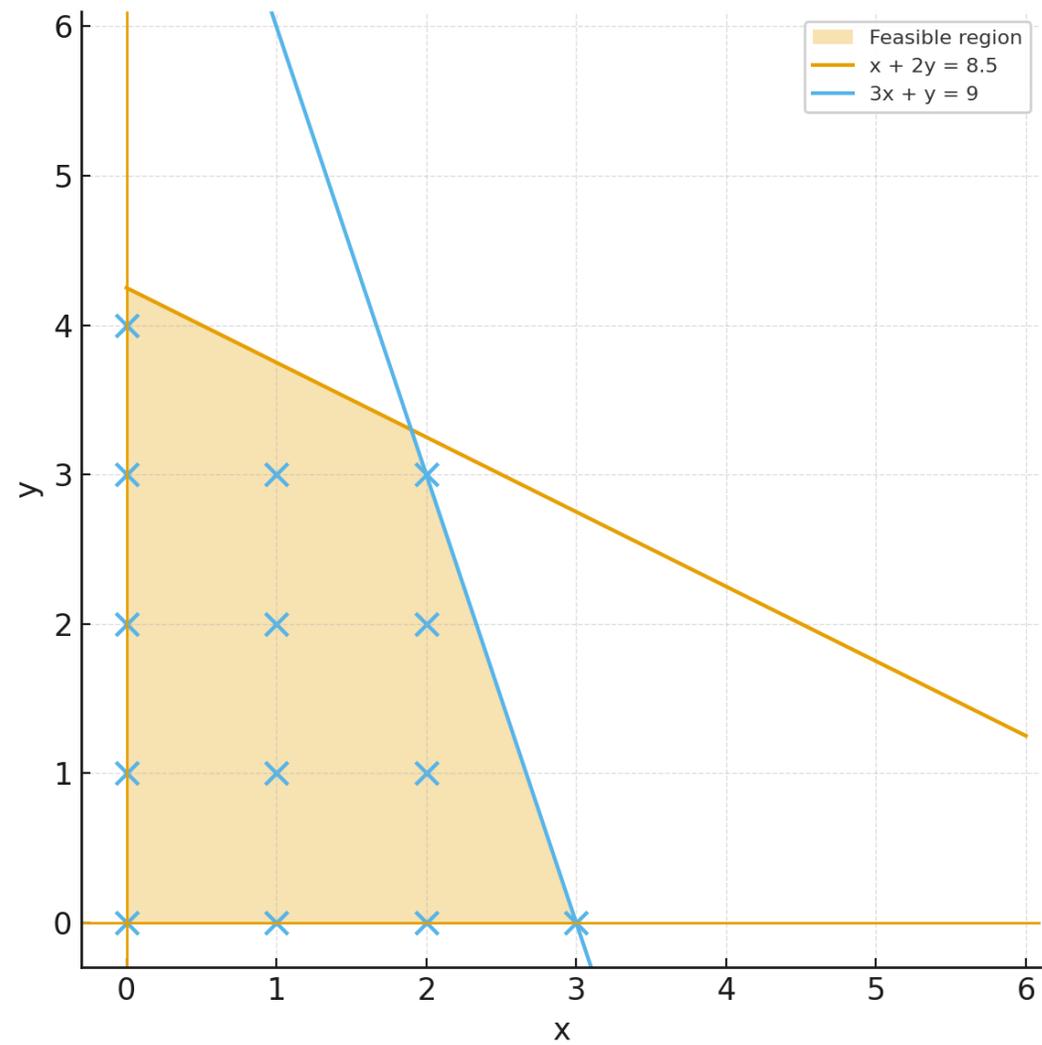
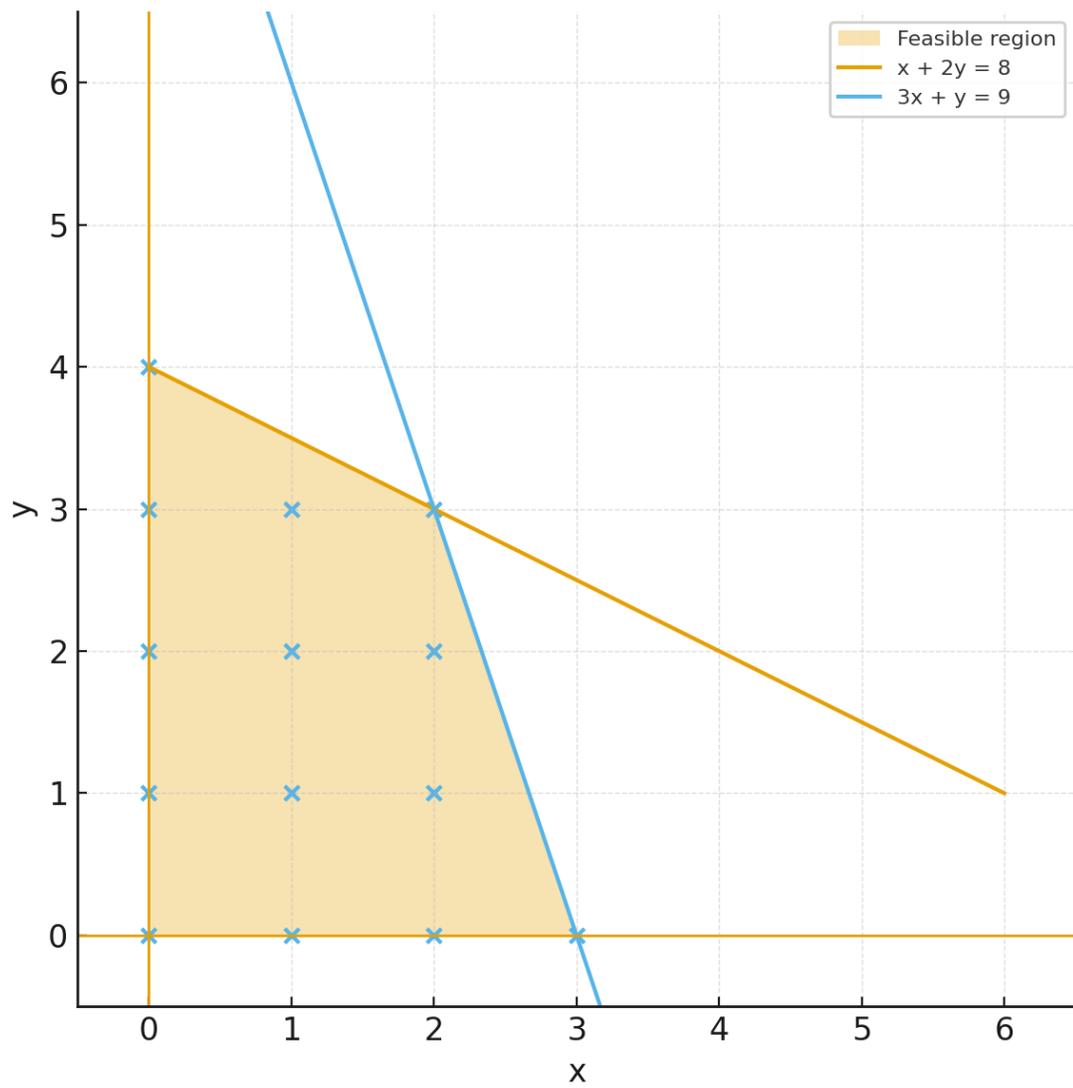
- Integer Programs we can solve efficiently, **optimally**



Today

- Integer Programs we can solve efficiently, **optimally**
- The optimal solution to a linear program will always lie at a “corner” of the feasible region
- AKA an **extreme point**
- All corners integral:
 - A “perfect formulation”
 - Integrality gap is 1





Main Theorem

Given a bipartite graph $G = (L \cup R, E)$ define the following Linear Program:

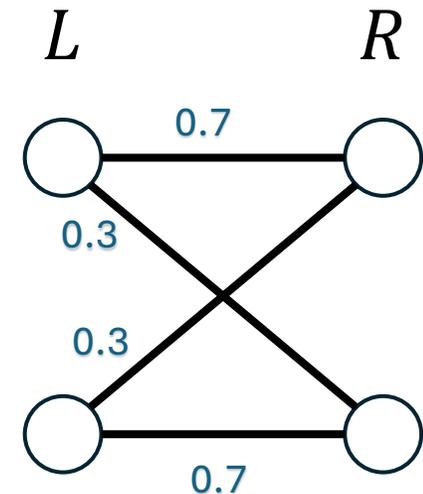
$$\min \sum_{e \in E} c_e x_e$$

such that:

$$\sum_{e \in \delta(u)} x_e = 1 \quad \text{for all } u \in L$$

$$\sum_{e \in \delta(v)} x_e = 1 \quad \text{for all } v \in R$$

$$x_e \geq 0 \quad \text{for all } e \in E$$



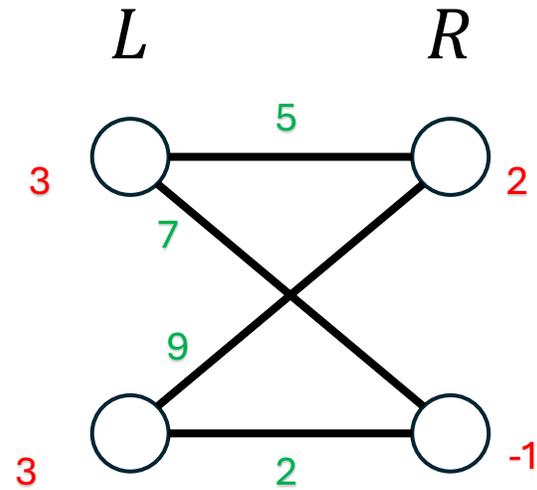
The above Linear Program is a **perfect formulation**.

Duality

$$\max \sum_{u \in L} p_u + \sum_{v \in R} p_v$$

such that:

$$p_u + p_v \leq c_e \quad \text{for all } e = (u, v) \in E$$



Seem familiar?

Duality

$$\min \sum_{e \in E} c_e x_e$$

such that:

$$\sum_{e \in \delta(u)} x_e = 1 \quad \text{for all } u \in L$$

$$\sum_{e \in \delta(v)} x_e = 1 \quad \text{for all } v \in R$$

$$x_e \geq 0 \quad \text{for all } e \in E$$

$$\max \sum_{u \in L} p_u + \sum_{v \in R} p_v$$

such that:

$$p_u + p_v \leq c_e \quad \text{for all } e = (u, v) \in E$$